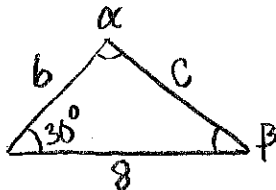1. The general area formula for a triangle is $A = \frac{1}{2}bh$, special cases are shown below:

   Case 1: SAS $A = \frac{1}{2}ab\sin\gamma \; ; \; \frac{1}{2}bc\sin\alpha \; ; \; \frac{1}{2}ac\sin\beta$

   Derivation:

   Find the area of the triangle: $a = 8, \; b = 6, \; \gamma = 30°$

   $$A = \frac{1}{2}ab\sin\gamma = \frac{1}{2}(8)(6)\sin 30°$$
   $$= \boxed{12 \text{ units}^2}$$

   Case 2: SSS   Heron's Formula $= A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{1}{2}(a+b+c)$

   Derivation: p. 551-552

   Find the area of the triangle: $a = 4, \; b = 5, \; c = 7$

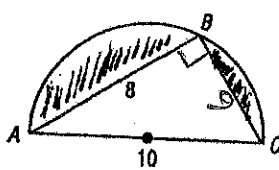 $S = \frac{1}{2}(a+b+c)$

   $S = \frac{1}{2}(4+5+7)$

   $S = 8$

   $A = \sqrt{8(8-4)(8-5)(8-7)}$

   $A = \sqrt{8(4)(3)(1)}$

   $A = \sqrt{96} = 4\sqrt{6} = \boxed{9.8 \text{ units}^2}$

2. Find the area of the shaded region enclosed in a semicircle of diameter 10 inches. The length of the chord AB is 8 inches. [Hint: Triangle ABC is A (R) $\triangle$]

   ① Find Area of the semicircle
   $$A = \frac{\pi r^2}{2} = \frac{\pi(5)^2}{2} = \frac{25\pi}{2} \text{ in}^2$$

   ② Find Area of $\triangle ABC \rightarrow \frac{1}{2}(6)(8) = 24 \text{ in}^2$

   ③ Area of shaded region = (area of semicircle) − (Area of $\triangle ABC$)
   $$A = \frac{25\pi}{2} \text{ in}^2 - 24 \text{ in}^2 = \boxed{15.27 \text{ in}^2}$$
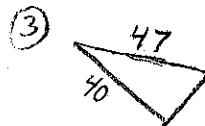
3. To approximate the area of a lake, a surveyor walks around the perimeter of the lake, taking measurements shown in the illustration. Using this technique, what is the approximate area of the lake?

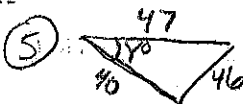   [Hint: use the Law of Cosines on the 3 triangles shown + then Find the sum of their Areas.]
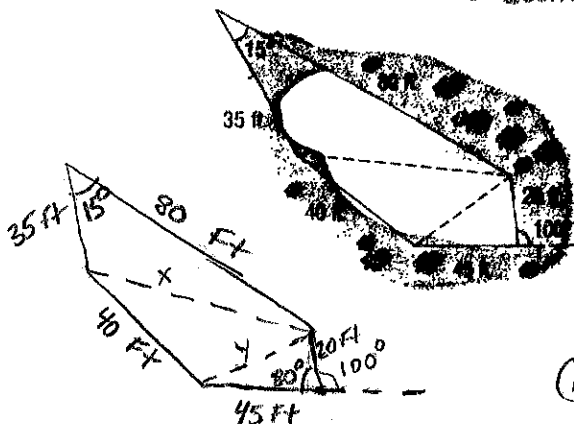
   ① $180 - 100 = 80°$

   ② $x = \sqrt{(35)^2 + (80)^2 - 2(35)(80)\cos(15°)}$
   $x \approx 47$

   ③ 47

   ④ $y = \sqrt{(45)^2 + (20)^2 - 2(45)(20)\cos(80°)}$
   $y \approx 46$

   ⑤ $\cos^{-1}\left[\frac{(47)^2 + (40)^2 - (46)^2}{2(47)(40)}\right] = 63.24° = Y$

   ⑥ Area of lake $= \frac{1}{2}[35 \cdot 80 \cdot \sin 15° + 40 \cdot 47 \sin 63.24° + 45 \cdot 20 \cdot \sin 80°]$

   Area of lake =